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Theory including future not excluded: Formulation of complex action theory II

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In Ref. [1] we have found errata. They are composed of two parts: one part is for the body, which is also explained in our recently published book [2], while the other part is for the appendix, which is mainly a result of the corrections to Ref. [3]. They do not influence the result of the manuscript. Rather, the latter part provides us a new additional result: the Schrödinger equation described with the Hamiltonian $\hat{H}_B$ has been derived for the future state $|B(t)\rangle$ via the Feynman path integral in the complex action theory.

In the fifth line below Eq. (5.8), where $f(DD^\dagger)^{-1}$ should have been replaced with $(f(D)f(D)^\dagger)^{-1}$, we have chosen $f(D)$ such that $(P^\dagger)^{-1}(f(D)f(D)^\dagger)^{-1}P^\dagger = F(\hat{H}^\dagger)$, which is rewritten as $(f(D)f(D)^\dagger)^{-1} = F(D^\dagger)$. However, this relation does not stand, because the left-hand side is Hermitian, while the right-hand side is not Hermitian. Accordingly, the expression $Q' = F(\hat{H}^\dagger)Q$ below Eq. (5.8), which was introduced based on the above relation, has to be corrected. In addition, the next statement, "$F(\hat{H}^\dagger)Q \simeq F(\hat{H}^\dagger_{\text{eff}})Q$ for the restricted subspace," is not right. This is because, for any reasonable function h and any state $|A(t)\rangle = \sum_i a_i(t)|\lambda_i\rangle$ that obeys the Schrödinger equation $i\hbar\frac{d}{dt}|A(t)\rangle = \hat{H}|A(t)\rangle$, the following relation holds for large $t - T_A$: $h(\hat{H})|A(t)\rangle \simeq h(\hat{H}_{\text{eff}} + iB\Lambda_A)|\tilde{A}(t)\rangle \equiv \tilde{h}(\hat{H}_{\text{eff}})|\tilde{A}(t)\rangle$, where we have used the automatic Hermiticity mechanism and introduced $|\tilde{A}(t)\rangle \equiv \sum_{i\in A} a_i(t)|\lambda_i\rangle$, $\Lambda_A \equiv \sum_{i\in A}|\lambda_i\rangle\langle\lambda_i|_Q$, and another function $\tilde{h}$ such that $\tilde{h}(\text{Re}\,\lambda_i) = h(\text{Re}\,\lambda_i + iB)$. Similarly, the statement "$Q_2 = F(\hat{H}^\dagger_{\text{eff}})Q$ for the restricted subspace" given in Eq. (5.6) has to be corrected.

To correct the above points, on behalf of $F(\text{Re}\lambda_i) = |b_i|^2$ and Eq. (5.6), we introduce functions G and $\tilde{G}$ such that $G(\text{Re}\,\lambda_i + iB) = \tilde{G}(\text{Re}\,\lambda_i) = b_i$, and express Q_2 as follows:

$$
\begin{aligned}
Q_2 &= \sum_{i\in A} |b_i|^2 |\lambda_i\rangle_B\, {}_B\langle\lambda_i| \\
&= \sum_{i\in A} G(\hat{H}_{\text{eff}} + iB\Lambda_A)^\dagger |\lambda_i\rangle_B\, {}_B\langle\lambda_i| G(\hat{H}_{\text{eff}} + iB\Lambda_A) \\
&= \tilde{G}(\hat{H}_{\text{eff}})^\dagger Q\Lambda_A \tilde{G}(\hat{H}_{\text{eff}}), \qquad (1)
\end{aligned}
$$

where, in the second and third equalities, supposing that $\text{Re}\,\lambda_i$'s are not degenerate, we have used $|\lambda_i\rangle_B = Q|\lambda_i\rangle$, and ${}_B\langle\lambda_i|G(\text{Re}\,\lambda_i + iB) = {}_B\langle\lambda_i|G(\hat{H}_{\text{eff}} + iB\Lambda_A)$ for $i \in A$. We note that

$Q\Lambda_A = Q\sum_{i\in A}|\lambda_i\rangle\langle\lambda_i|_Q$ is Hermitian, and so is Q_2. Next we define Q' by $Q' \equiv G(\hat{H})^\dagger Q G(\hat{H}) = ({P_{G^{-1}}}^\dagger)^{-1}{P_{G^{-1}}}^{-1}$, where $P_{G^{-1}} \equiv G(\hat{H})^{-1}P$ diagonalizes $\hat{H}$: $(P_{G^{-1}})^{-1}\hat{H}P_{G^{-1}} = P^{-1}\hat{H}P = D$. In addition, we introduce $|\lambda_i\rangle^{G^{-1}} \equiv G(\hat{H})^{-1}|\lambda_i\rangle$, so that $|\lambda_i\rangle^{G^{-1}}$ is Q'-orthogonal, i.e., orthogonal with regard to the proper inner product $I_{Q'}$: $I_{Q'}(|\lambda_i\rangle^{G^{-1}}, |\lambda_j\rangle^{G^{-1}}) \equiv {}^{G^{-1}}\langle\lambda_i|Q'|\lambda_j\rangle^{G^{-1}} = \delta_{ij}$. We use the automatic Hermiticity mechanism for large $t - T_A$. Then, since $|A(t)\rangle$ behaves as $|\tilde{A}(t)\rangle \equiv \sum_{i\in A} a_i(t)|\lambda_i\rangle$, Q' used in the normalized matrix element $\langle\mathcal{O}\rangle_{Q'}^{AA}$ is estimated in the subspace restricted by A as follows:

$$
\begin{aligned}
Q' &\simeq G(\hat{H}_{\text{eff}} + iB\Lambda_A)^\dagger Q\Lambda_A G(\hat{H}_{\text{eff}} + iB\Lambda_A) \quad \text{for the restricted subspace} \\
&= \tilde{G}(\hat{H}_{\text{eff}})^\dagger Q\Lambda_A \tilde{G}(\hat{H}_{\text{eff}}) \\
&= Q_2, \qquad (2)
\end{aligned}
$$

where in the last equality we have used Eq. (1). The three sentences "We first point out … replaced with $|\tilde{A}(t)\rangle$" below Eq. (5.8) should be replaced with the above argument.

A dt-dependent normalization factor, say $\frac{1}{\alpha(dt)}$, should be inserted on the right-hand sides of Eq. (A.2) and of the first line of Eq. (A.4). The following sentence should be inserted after the sentence "C is an arbitrary … complex plane" below Eq. (A.2): "In addition, $\alpha(dt)$ is a dt-dependent normalization factor, which is properly fixed later." The factor $\sqrt{\frac{2\pi i\hbar dt}{m}}$ in the second line of Eq. (A.4) should be deleted. The following sentences should be inserted after the phrase "where … Eq. (3.7)" below Eq. (A.4): "Here we have taken $\alpha(dt) = \sqrt{\frac{2\pi i\hbar dt}{m}}$ so that both sides of Eq. (A.4) correspond to each other in the vanishing limit of dt. Then Eq. (A.4) is reduced to $|\psi(t+dt)\rangle = e^{-\frac{i}{\hbar}\hat{H}dt}|\psi(t)\rangle$." The next sentence, "Thus we have found that … Eq. (A.2)," below Eq. (A.4) should be replaced with "Thus we have derived the Schrödinger equation and found that … Eq.(A.2)." The following sentence should be added after the above replaced sentence: "Such a derivation of the Schrödinger equation is well known in the real action theory [4]." Factors $\frac{1}{\alpha(dt)^*}$, $\frac{1}{\alpha(-dt')^*}$, and $\frac{1}{\alpha(-dt)}$ should be inserted on the right-hand side of the equation in the second sentence of the last paragraph of the appendix, on the right-hand sides of Eqs (A.5) and (A.6), respectively. The second sentence below Eq. (A.6), "Indeed, $\hat{H}_B$ is given … $\hat{H}^\dagger$," should be replaced with "Indeed, we obtain the Schrödinger equation $|B(t-dt)\rangle = e^{\frac{i}{\hbar}\hat{H}_B dt}|B(t)\rangle$, where $\hat{H}_B$ is given … $\hat{H}^\dagger$."

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